P425/1
PURE
MATHEMATICS
Paper 1
July /Aug. 2019
3 hours

KAMULI JOINT EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *questions in* ***section A*** *and any* **five** *from* ***section B.***

All necessary working must be shown clearly.

Silent non – programmable scientific calculators and mathematical tables may be used.

Any extra question(s) attempted in section ***B*** *will* **not** *be marked.*

SECTION A (40 MARKS)

1. Express $\boldsymbol{\cos x + 3\sin x}$ in the form $\boldsymbol{R\cos(x - \alpha)}$. Hence solve the equation $\boldsymbol{\cos 2\theta + 3\sin 2\theta = 2}$ for $0^0 < \theta < 90^0$ ***(05 marks)***
2. A line $\boldsymbol{2x - y + 3 = 0}$ touches a circle whose Centre is **(-4,5)**. Determine the equation of the circle. ***(05 marks)***
3. Solve the following simultaneous equations.

$$\boldsymbol{x + 3y + 2z + 13 = 0}$$
$$\boldsymbol{2x - 6y + 3z = 32}$$
$$\boldsymbol{3x - 4y - z = 12}$$
(05 marks)

4. In the equation $px^2 + qx + r = 0$, one of the roots is the square of the other. Without solving the equation, show that $q^3 = pr(3q - p - r)$. ***(05 marks)***
5. Find $\displaystyle\int \frac{\mathbf{1}}{\mathbf{1+\sin x}}\boldsymbol{dx}$ ***(05 marks)***
6. By use small changes, show that $\boldsymbol{\sqrt[5]{244} = 3\frac{1}{405}}$ ***(05 marks)***
7. If the position vectors of the points P and Q are $2i + 4j + 6k$ and $-3i + 2j + 8k$ respectively, find the position vector of the point M which divides PQ externally in the ratio 5:3 . ***(05 marks)***
8. Find the coefficient of x^{17} in the expansion of $\left(x^3 + \frac{1}{x^4}\right)^{15}$ ***(05 marks)***

SECTION B (60 MARKS)

9. (a) If $\tan X = a$, $\tan Y = b$, $\tan Z = c$. Prove that $\tan(X + Y + Z) = \frac{a+b+c-abc}{1-ab-ac-bc}$. Hence show that $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{\pi}{4}$. (***06 marks***)

   (b) Show that if $\sin(x + \alpha) = k\sin(x - \alpha)$ then $\tan x = \frac{(k+1)}{(k-1)}\tan\alpha$. Hence solve the equation $\sin(x + 20^0) = 2\sin(x - 20^0)$ for $0^0 \le x \le 180^0$. (***06 marks***)

10. (a) Using Maclaurin's theorem, determine the first three non-zero terms of the series for $\log_5(1 + e^x)$. (***06 marks***)

    (b) Use binomial theorem to obtain the first four terms of the expansion $\sqrt[4]{(1 - 16x)}$. Hence find $(39)^{\frac{1}{4}}$ correct to 5s.f (take $x = \frac{1}{10000}$). (***06 marks***)

11. (a) Find the perpendicular distance of a point (3,0,1) from the line $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z}{12}$.

(b) Find the Cartesian equation of the plane through points A(2,-1,2), B(0,3,-4) and C(7,4,-1). **(07 marks)**

12. Express $\frac{x+1}{x^2(x^2+1)}$ as partial fractions. Hence evaluate $\int_1^2 \frac{x+1}{x^2(x^2+1)} dx$ **(12 marks)**

13. (a) Given that $Z = \frac{(1-i)(\sqrt{3}-i)}{(1-i\sqrt{3})}$, express Z in polar form. **(04 marks)**

(b) Show that the locus of $\left|\frac{Z-1}{Z+1}\right| = 2$ is a circle. State its centre and radius. (**04 marks**)

(c) Solve the equation $Z^2 - 4(1+i)Z + 9 + 8i = 0$. **(04 marks)**

14. (a) Given that x is a real number, prove that the function $y = \frac{(x+1)(x-3)}{x(x-2)}$ does not lie between 1 and 4.

(b) Determine the turning point(s) and distinguish between them.

(c) State the equations of the asymptotes and the points at which the curve cuts both axes.

(d) Sketch the curve. **(12 marks)**

15. (a) The normal to the parabola $y^2 = 4ax$ at the point A$(at^2, 2at)$ meets the axis of the parabola at T and TA is produced beyond A to B so that $\overrightarrow{TA} = \overrightarrow{AB}$. Show that the equation of the locus of B is $y^2 = 16a(x + 2a)$. (**06 marks**)

(b) Prove that the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2}$=1 at the point $(a\sec\theta, b\tan\theta)$ is $by + ax\sin\theta = (a^2 + b^2)\tan\theta$. If the normal meets the x-axis at P and the y-axis at Q, find the locus of the mid-point of PQ. **(06 marks)**

16. (a) Solve the differential equation $(x + y)\frac{dy}{dx} = x - y, y(3) = -2$. **(05 marks)**

(b) The rate at which a disease spreads through a certain community is found to be directly proportional the fraction x of the community infected after t months but inversely proportional to the fraction not yet infected.

(i) Form a differential equation connecting x and t.

(ii) Show that the general solution to the equation can be expressed as $e^{kt} = Axe^{-x}$, where k and A are constants. When first noticed, one half of the community was infected and by this instant the disease is spreading at a fraction $\frac{1}{4}$ per month, show that the particular solution to the differential equation is $e^{t} = 16x^4e^{2-4x}$**(07 marks)**

END